

TABLE 6.—Flood stages in the Mississippi River drainage, except the Ohio River, March, 1917.

River.	Station.	Flood stage.	Above flood stages—dates.		Crest.	
			From—	To—	Stage.	Date.
		<i>Fect.</i>			<i>Fect.</i>	
Mississippi	Hannibal, Mo.	13			12.9	31
Do.	New Madrid, Mo.	34	17	(*)	38.6	26-28
Do.	Memphis, Tenn.	35	22	(*)	38.6	31
Do.	Helena, Ark.	42	23	(*)	47.2	31
Do.	Arkansas City, Ark.	42	21	(*)	47.5	31
Do.	Greenville, Miss.	42			40.0	31
Do.	Vicksburg, Miss.	45			43.9	31
Yazoo	Swan Lake, Miss.	25	18	(*)	28.2	31
Do.	Yazoo City, Miss.	25			23.5	31
Cedar	Cedar Rapids, Iowa	14	26	27	17.3	27
Illinois	Peoria, Ill.	16			15.6	21
Do.	Beardstown, Ill.	12	17	(*)	13.6	23-26
CACHE	Jelks, Ark.	9			8.9	23-25
James	Huron, S. Dak.	9	25	(*)	12.0	31
Floyd	Merrill, Iowa.	13	22	23	14.5	22

\* Continued above flood stage after end of month.

## A SKEW FREQUENCY CURVE APPLIED TO STREAM GAGE DATA.

551.482

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[Dated U. S. Office of Farm Management, Washington, Mar. 7, 1917.]

From a study of the frequency distributions of rainfall amounts it has appeared that nearly all the records show skew frequency polygons, in which the median observation has a smaller value than the mean of all the observations.<sup>1</sup> Similar skewness exists in the frequency distribution of daily gage heights of the upper Paraná at Posadas (Misiones) Argentina ( $\phi = 27^{\circ} 24' S.$ ,  $\lambda = 55^{\circ} 50' W.$ ,  $H = 138$  m.). Dr. Wolff, Chief of the Oficina Hidrométrica of Argentina, has computed the frequencies of these gage heights for the 12 years 1904-1915, representing a total of 4,383 observations.<sup>2</sup> His frequency polygon closely resembles those obtained for rainfall amounts. The data have been used in constructing figure 1, which is the frequency polygon and the skew frequency curve for the limiting case computed in the manner suggested by Tolley.

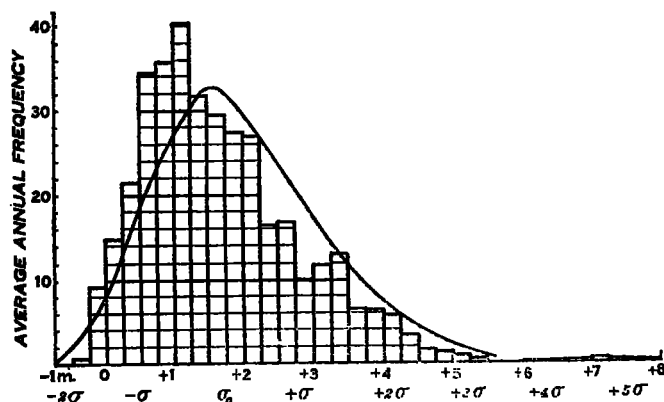


FIG. 1.—Frequency polygon and skew frequency curve for the limiting case for gage heights on the Paraná at Posadas.

Although Wolff has calculated the standard deviation, he has made little use of it in his tabulation. It may be of interest to carry the computation a little further.

<sup>1</sup> See Tolley, H. R. "Frequency curves of climatic phenomena." MONTHLY WEATHER REVIEW, Nov. 1916, 44: 634-642.

<sup>2</sup> Wolff, E. In República Argentina, Oficina meteorológica nacional, Boletín mensual, Buenos Aires, no. 3, marzo 1916, 1: 40-41.

Using the notation suggested by Tolley, the following values have been obtained:

Quantity.	Symbol.	Value.
Number of observations.....	$n$	4383*
Mean gage height.....	$M_o$	$5.692 \frac{\text{meters}}{4}$ *
Convenient number near the mean.....	$M$	$1.620 \frac{\text{meters}}{4}$ *
Departure from $M$ .....	$d$	
Sum of departures.....	$\Sigma d$	$+93 \frac{\text{meters}}{4}$ *
Average departure from $M$ .....	$\Sigma d/n$	$+0.2124 \frac{\text{meters}}{4}$ *
Sum of squared departures from $M$ .....	$\Sigma d^2$	$+90,293 \frac{\text{meters}}{4}$ *
Average square of departures from $M$ .....	$\Sigma d^2/n$	$+20.6007 \frac{\text{meters}}{4}$ *
Average square of departures from the mean.....	$\Sigma d^2/n - (\Sigma d/n)^2$	$+20.557 \frac{\text{meters}}{4}$ *
Standard deviation.....	$\sigma = \sqrt{[\Sigma d^2/n - (\Sigma d/n)^2]}$	$4.525 \frac{\text{meters}}{4}$ *
Sum of cubed departures from $M$ .....	$\Sigma d^3$	$+128,063 \frac{\text{meters}}{4}$
Average cube of departures from $M$ .....	$\Sigma d^3/n$	$+97,662 \frac{\text{meters}}{4}$
Average cube of departures from the mean.....	$\mu_3 = \Sigma d^3/n - 3(\Sigma d/n)(\Sigma d^2/n) + 2(\Sigma d/n)^3$	$+84,580 \frac{\text{meters}}{4}$
Constant representing the value of skewness.....	$k = \frac{\mu_3}{\sigma^3}$	$+0.913$

\* Values computed by Wolff.

The value of  $k$  (+0.913) is of the same sign and the same order of magnitude as that generally found in the case of rainfall amounts. This is to be expected, as variations in stream flow are closely dependent on variations in rainfall. It would be interesting to determine the variations in rainfall over the Paraná drainage basin above Posadas, but this is scarcely practicable owing to the vast extent of the area and the paucity of usable records.

In his statistical computation Wolff—like the North American engineers who have done similar work, notably Allen Hazen,<sup>3</sup> of New York, and R. W. Davenport, of the U. S. Geological Survey—has tacitly assumed that the standard deviation defines the frequency distribution, and has not computed any measure of skewness. He has, however, clearly shown by his polygon (fig. 1) that skewness exists and has not attempted to bring the tails of his polygon, or the summation curve, into arbitrary agreement with the normal curve. Tolley has shown that if skewness of the order here under consideration exists, its amount must be determined by the use of moments higher than the second,  $\sigma$ , if predictions based on the limiting case are to have value. He has also shown that the measure of skewness,  $k$ , is determined with as great accuracy as is warranted by the ordinary record when the third moment,  $\mu_3$ , is used.

The results of this tabulation of the Posadas data furnish a further indication that meteorological and allied phenomena tend to follow definite frequency distributions which can be investigated by modern statistical methods. The application of these methods to stream flow data is obviously of great importance in river and flood studies as well as in irrigation and other investigations upon which depend the utilization of arid and semi-arid regions.

The skewness in the Posadas frequency distribution shows the advisability of using the third moment in the

<sup>3</sup> See especially Storage to be provided in impounding reservoirs for municipal water supply. Trans., Am. soc. c. e., New York, 1914, 77: 1539-1669.

computation of the limiting case. Wolff has not attempted to determine this limiting case nor has he computed the third moment. He has used the second moment only incidentally in determining what he calls a "coefficient of variation," which is not, however, the same quantity as is generally understood by the term. Wolff's "coefficient of variation" is defined as

$$\frac{2\sigma}{s} = \frac{2 \times 1.312 \text{ meters}}{3.993} = 0.567 \text{ meters,}$$

where  $\sigma$  = standard deviation

and  $s = h_0 \text{ max} - h_0 \text{ min}$

when  $h_0 \text{ max}$  = mean of the annual maxima of gage heights

$h_0 \text{ min}$  = mean of the annual minima of gage heights

$M$  = mean gage height

$\eta_2 = M + \frac{1}{2}(h_0 \text{ max} - M)$

$\eta_1 = M - \frac{1}{2}(M - h_0 \text{ min})$

It is easily seen that  $\eta_2$  represents a gage height midway between  $M$  and  $h_0 \text{ max}$ , and  $\eta_1$  a height midway between  $M$  and  $h_0 \text{ min}$ .

Stages above  $\eta_2$  are designated as "high-water stages"; they obtained on an average of 55 days a year (1904-1915). Stages below  $\eta_1$  are "low-water stages"; they obtained on an average of 104 days a year. Stages between these limits are "ordinary stages"; they obtained on an average of 206 days a year (1904-1915).

Although the published material is wholly tabular and graphic, the frequency polygon shows clearly the occurrence of different stages of the Paraná; this polygon, while skewed, is regular without breaks, and the tables

furnish data which may be studied by modern statistical methods. If these methods are applied to stream-flow data, it seems probable that the average frequencies of various stages can be determined in the limiting case and these determinations should be of value in studies of floods, water supply, and water rights.

#### MEAN LAKE LEVELS DURING MARCH, 1917.

By UNITED STATES LAKE SURVEY.

[Dated: Detroit, Mich., Apr. 5, 1917.]

The following data are reported in the "Notice to Mariners" of the above date:

Data.	Lakes.*			
	Superior.	Michigan and Huron.	Erie.	Ontario.
Mean level during March, 1917:				
Above mean sealevel at New York.....	<i>Fcet.</i> 602.33	<i>Fcet.</i> 580.46	<i>Fcet.</i> 571.53	<i>Fcet.</i> 245.17
Above or below—				
Mean stage of February, 1917.....	-0.00	-0.03	+0.18	+0.09
Mean stage of March, 1916.....	+0.18	+1.02	-0.34	-0.29
Average stage for March, last 10 years.....	+0.76	+0.55	-0.26	-0.68
Highest recorded March stage.....	+0.05	-2.49	-2.32	-2.64
Lowest recorded March stage.....	+1.67	+1.53	+0.70	+0.87
Average relation of the March level to—				
February level.....	-0.2	±0.0	+0.1	+0.2
April level.....	±0.0	-0.2	-0.6	-0.5

\*Lake St. Clair's level: In February=574.87; March=574.79.